July 26, 2023

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Design Task Group Update:

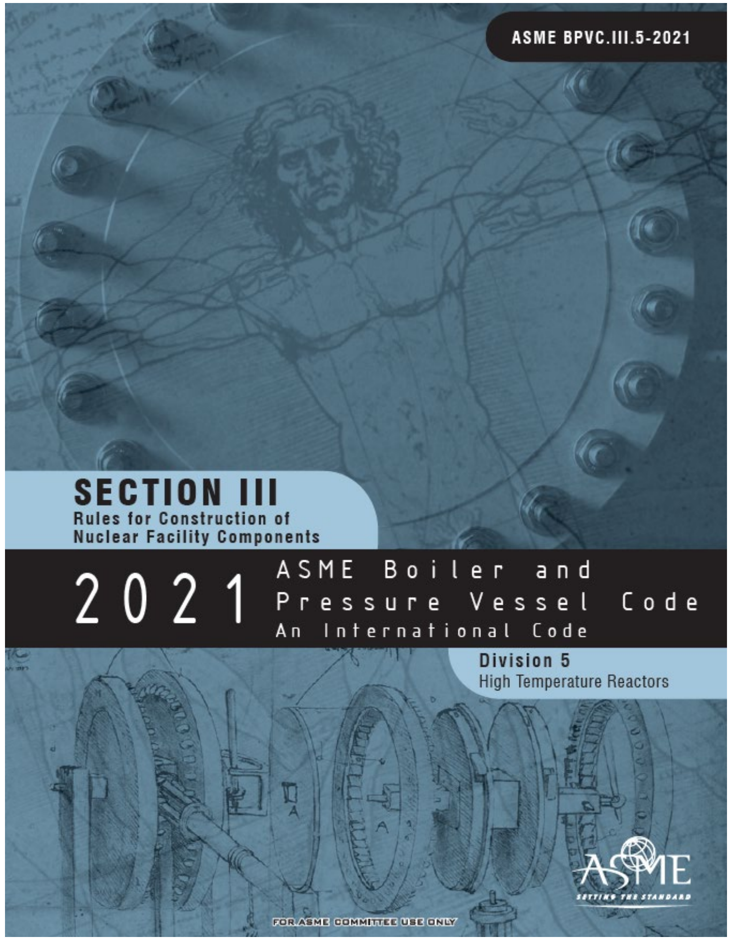
Activities Jan. 2023 – May 2023

DOE ART Gas-Cooled Reactor (GCR) Review Meeting
Virtual Meeting
July 25 – 27, 2023

ART
Advanced Reactor Technologies

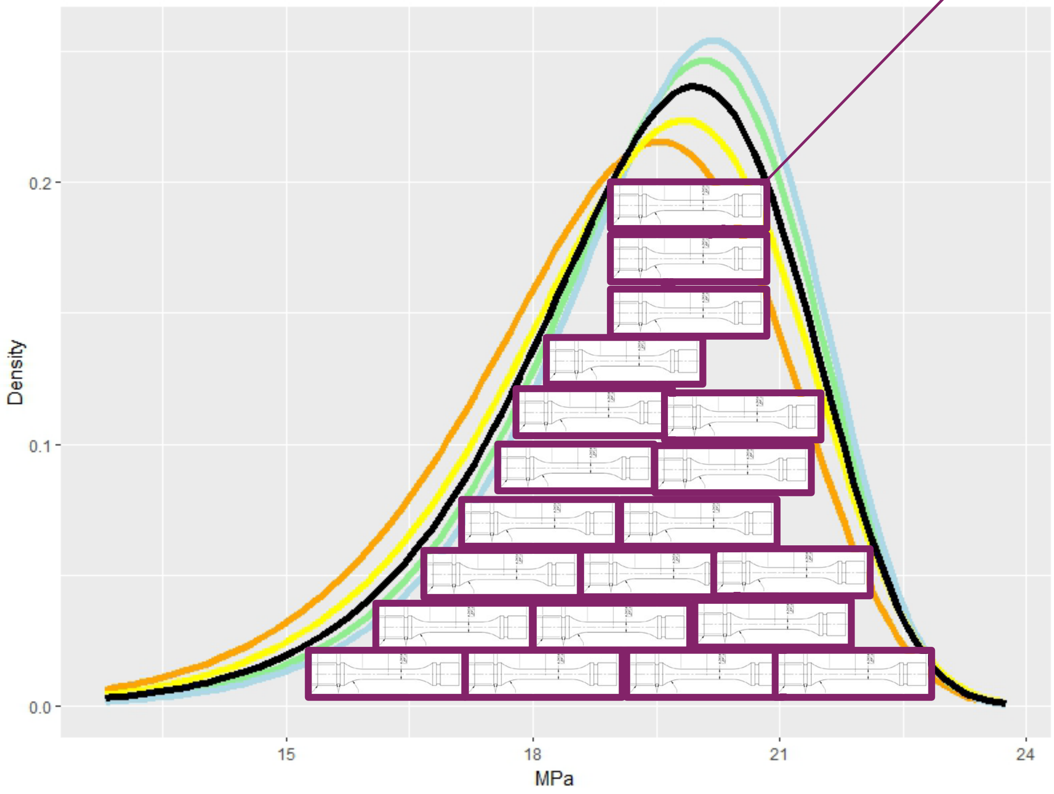
Outline of presentation

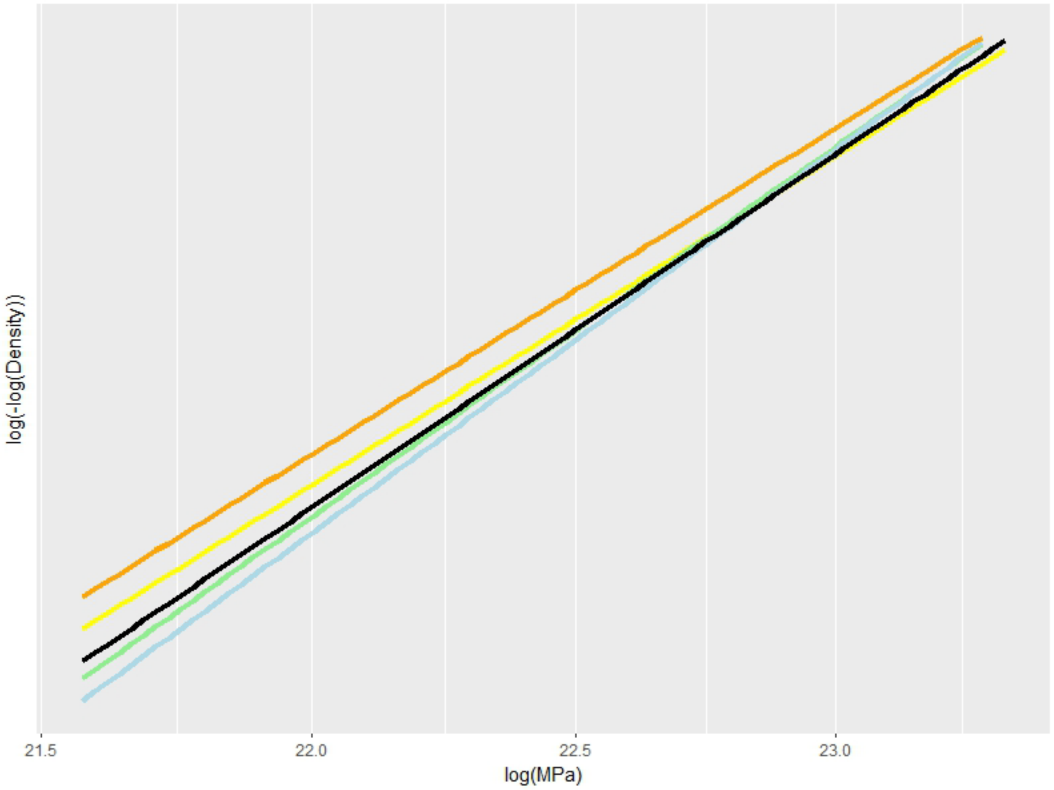
- Background
- Full Assessment
  - Disparate flaw distribution
  - Tuning V_m and Δ
  - Mesh refinement
  - Location
  - Sample size requirements
  - Margin
- Simplified Assessment
  - Stress terminology
  - R_{tf}
- Schedule to complete goal (2025 BPVC)
- Conclusions



Background – Weibull tensile strength distribution for quasi-brittle materials

N=288

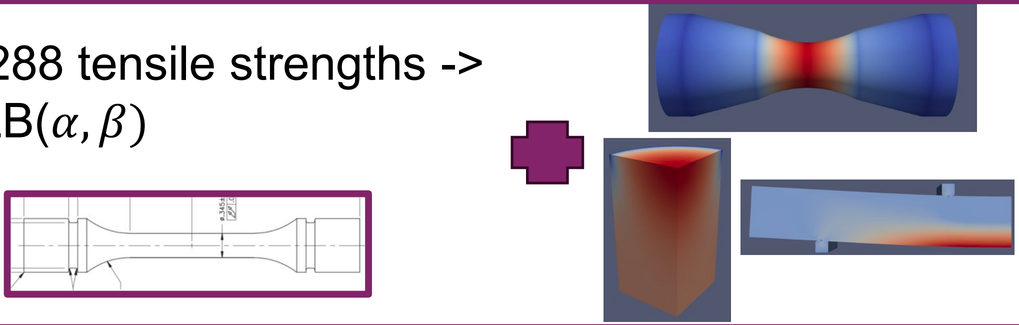




Weibull CDF: $F(\sigma|\alpha,\beta,\mu) = 1 - e^{-\left(\frac{\sigma-\mu}{\beta}\right)^{\alpha}}$

Assessments

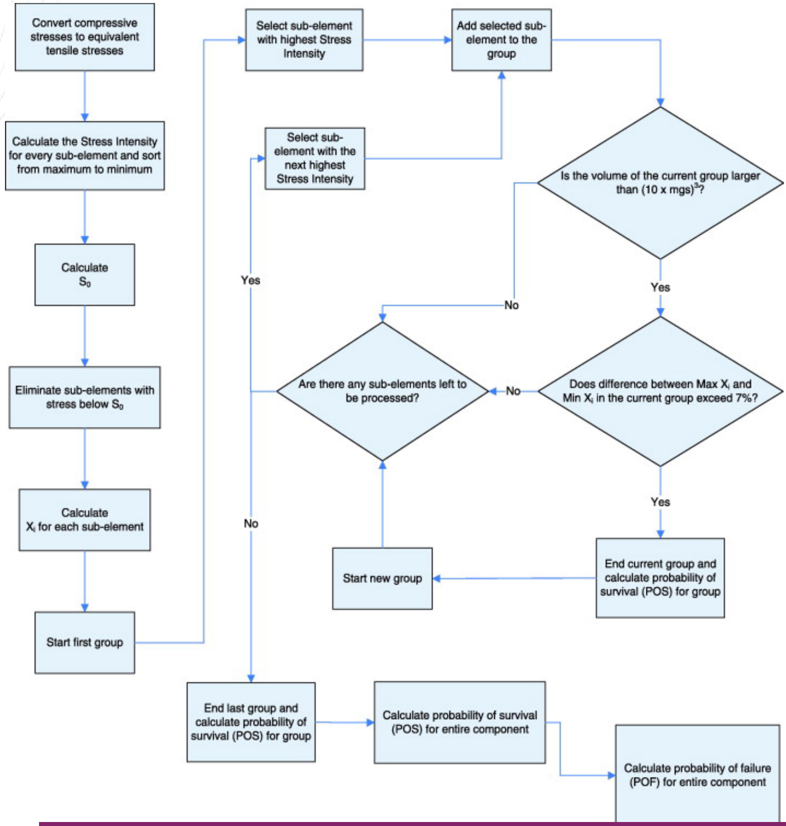
N=288 tensile strengths -> μ, LB(α, β)



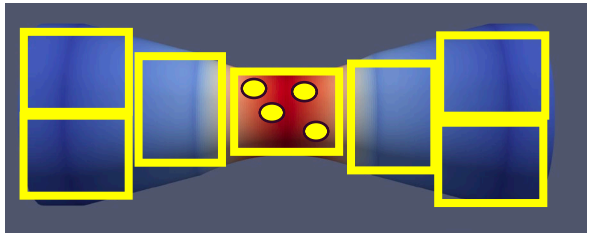
Full

Simplified

Inputs: 3-parameter Weibull lower bounds, Vm, Δ, entire component equivalent stress distribution



Uses weakest link theory



Output: Probability of Failure

Inputs: 2-parameter Weibull lower bounds, R_{tf}, membrane stress and peak equivalent stress from FEA output

1. Using lower bounds for parameters, invert the Weibull CDF: $S_g(10^{-4}) =$

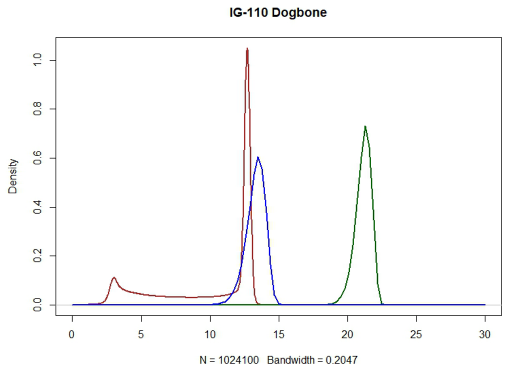
$$\beta(-\ln(1-10^{-4}))^{\frac{1}{\alpha}}$$

1. Calculate the limits for both checks:
   1. $S_g(10^{-4})$ is defined as the stress associated with the 10^{-4} quantile of the 2-parameter Weibull lower bound distribution.
   2. $R_{tf} * S_g(10^{-4})$ is the ratio of the flexural mean strength to the tensile mean strength (R_{tf}) times the allowable stress, resulting in an allowable stress value.
2. Calculate C_m and PES
3. Check 1: $C_m < S_g(10^{-4})$
4. If component passes Check 1, perform Check 2: $PES < R_{tf} * S_g(10^{-4})$

Output: Allowable tensile stress ($S_g(10^{-4})$) and allowable flexural stress ($R_{tf} * S_g(10^{-4})$), assuming component of SRC-1

Full Assessment: Disparate flaw distribution

Currently HHA-3217 has an engineered threshold reduction step for when there is too much separation between the lower bound strength distribution and the FEA stress output. The shape parameter depends on the threshold parameter, and it has been documented in the literature, as well as shown by the TG with the BP data, that the full assessment can become more conservative than the simplified when the threshold is reduced but the shape parameter is not updated, contrary to the spirit of the Code. A record was submitted twice to make update the shape parameter when the threshold is reduced, and disapproved. The most recent ballot was disapproved by just one, requesting that we demonstrate the full assessment would still be "conservative enough". Upon looking into this further, we determined that not updating the shape parameter may have been intentional, to better capture the disparate flaw distribution. However, the problem still exists that not updating the shape parameter makes the full assessment more conservative than the simplified. Both cannot happen – the disparate flaw distribution cannot be captured as it currently is while ensuring the simplified assessment is always more conservative. The WG-NMD needs to decide the next steps to take.



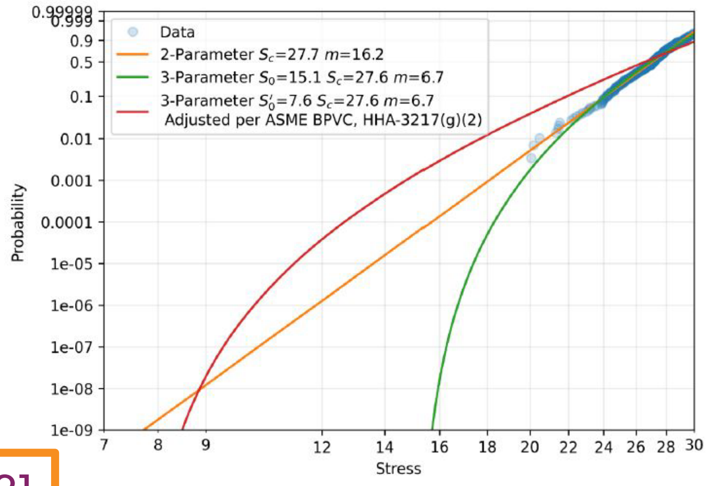


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Figure 4-2. Extension of Figure 4-1

- US experiments, H451 (Kennedy and Eatherly, 1986)

Figures copied from:

[1] https://www.osti.gov/servlets/purl/5283970

[2] https://nucleus.iaea.org/sites/graphiteknowledgebase/reports/INGSM/INGSM-8%20Material/3_22.ppt

[3] Saitta, M. (2023). Justification for record 21-1581: adjusting shape parameter. Memo.

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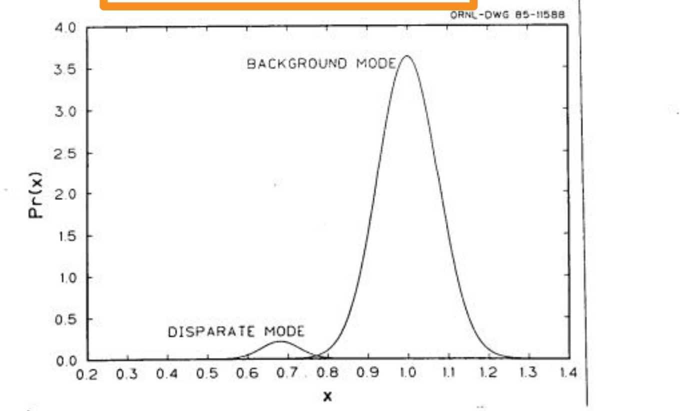
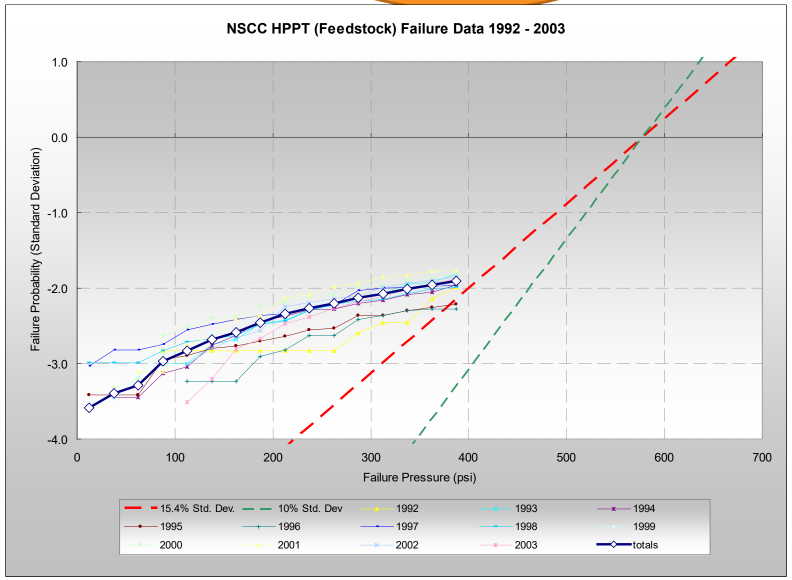
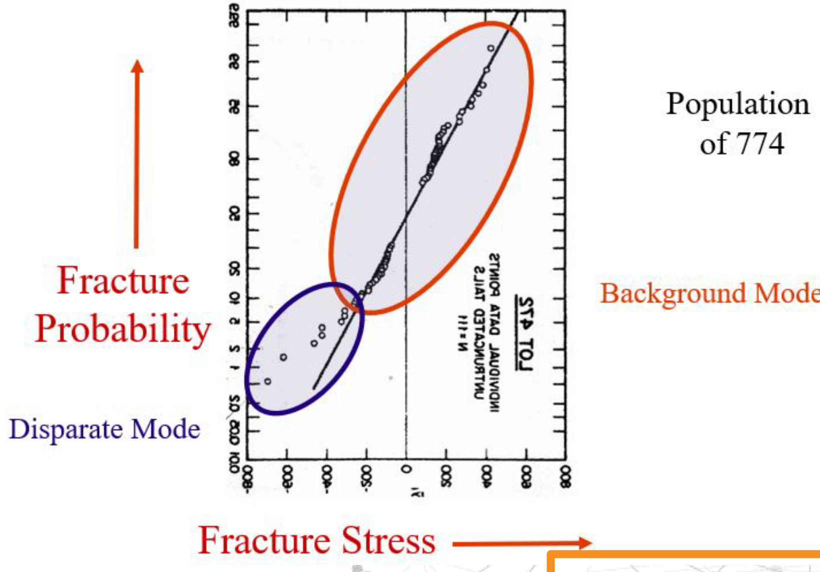


Fig. 6. Plots of normal distributions corresponding to the bimodal model normalized to the mean strength of the background flaw distribution. The disparate mode represents 3.4% of the total distribution.





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Full assessment: Tuning V_m and Δ (1/3)

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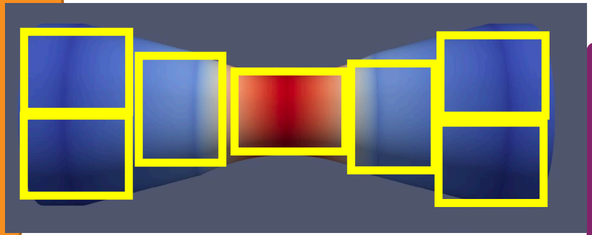
PROBLEM

- The 2021 grouping criteria, based on fracture toughness, increased the minimum link volume for fine grain graphites relative to the 2019 grouping criteria, based on grain size, as intended.
- However, it decreased the minimum link volume for medium grain graphites, adding unnecessary conservatism.
- The 2021 Vm equation was wrong and the corrected equation creates too small of groups. Therefore, we are not trying to correct the 2021 erroneous equation.



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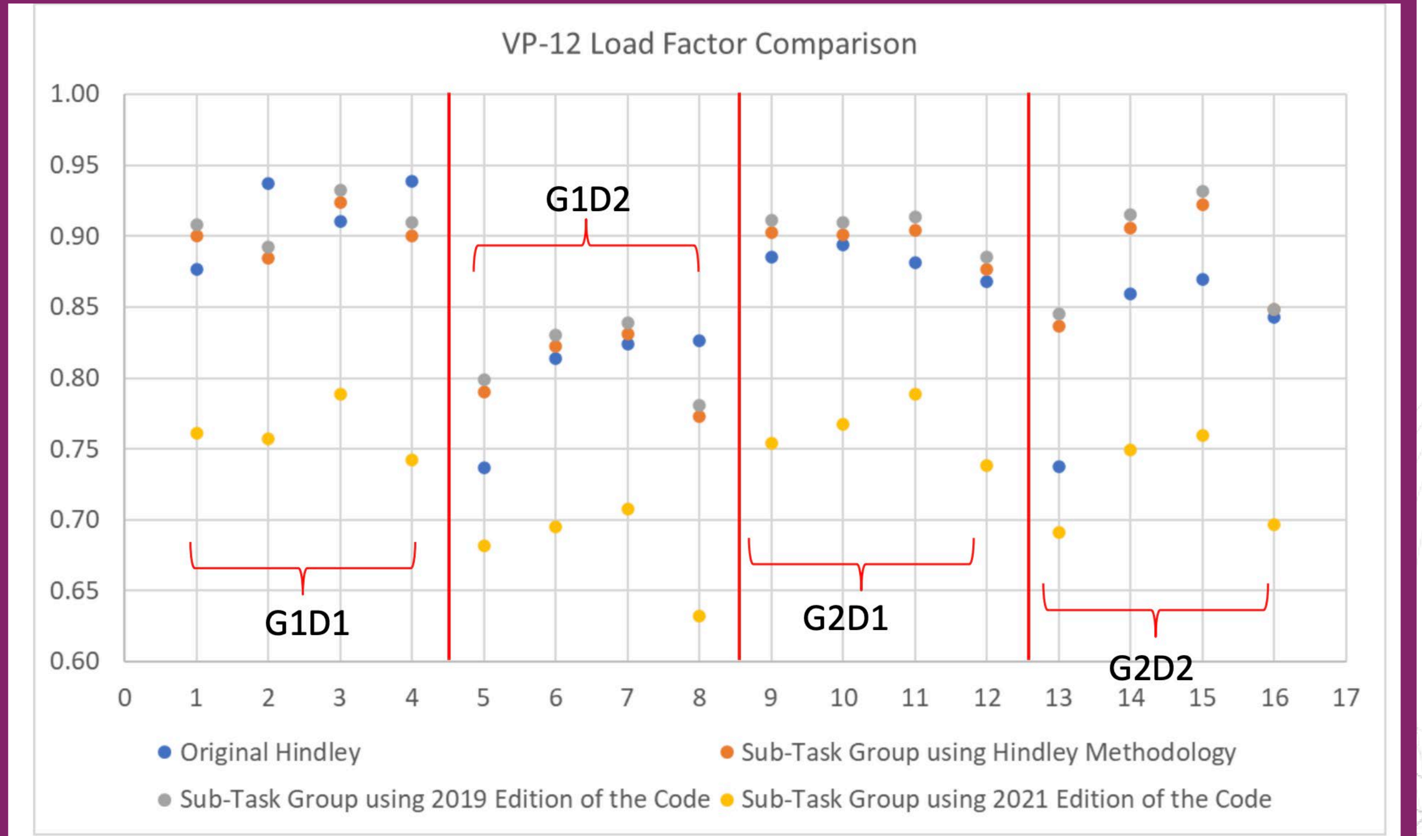
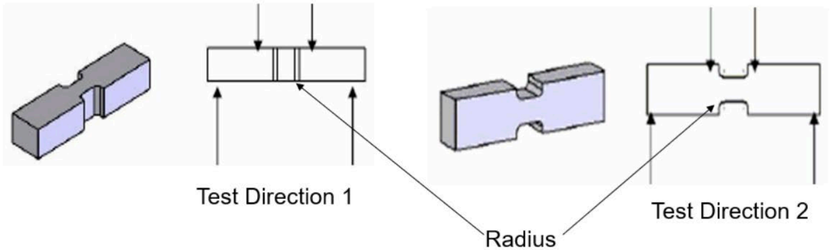


A Load Factor of 1 indicates the full assessment methodology is perfectly estimating the median probability of failure. As the Load Factor decreases from 1 to 0, the the full assessment becomes more conservative. The G1D1-G2D2 geometries were made from NBG-18.

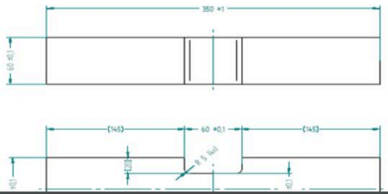
[4] Saitta, M., Beirnaert, G. , Quick, J. , Burns, Z., Hoffman, W., & Mack, A. (2023). Tuning of Vm and Delta. ASME Boiler and Pressure Vessel Code Week May 2023, Las Vegas, NV. Design Task Group Meeting.

Full Assessment: tuning V_m and Δ (2/3) GEOMETRY DRAWINGS applied to 2114, IG-110, NBG-18, NBG-17, and PCEA

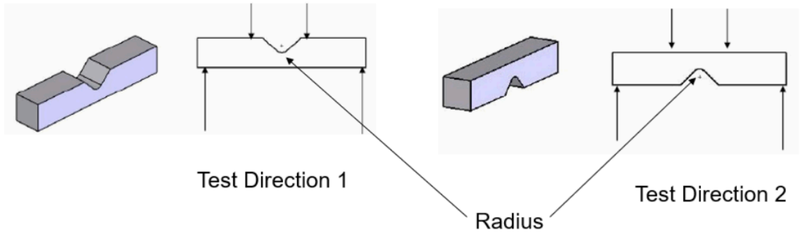
VP-12 – Geometry 1 with 2 Test Directions



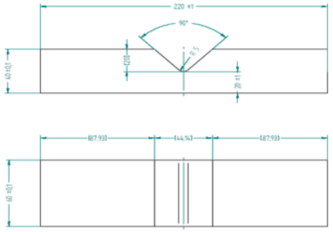
Radius [mm]
1
5
10
20



VP-12 – Geometry 2 with 2 Test Directions



Radius [mm]
1
5
10
20



Hindley's Dogbone

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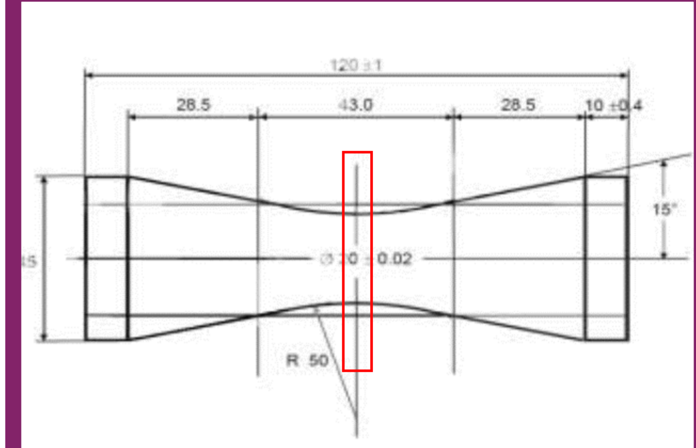
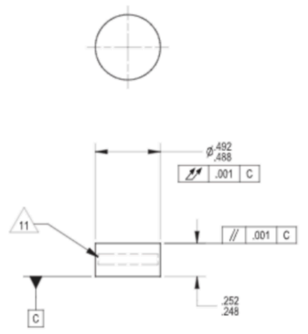


Baseline Program Model Geometry

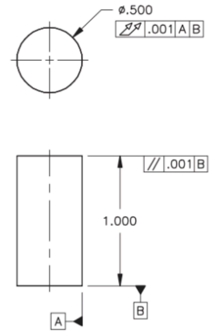
Grades considered: NBG18, NBG17, PCEA, IG110, 2114

- ***NBG18 test specimens are larger than other grades**
- **Dimensions shown for NBG17, PCEA, IG110, 2114**

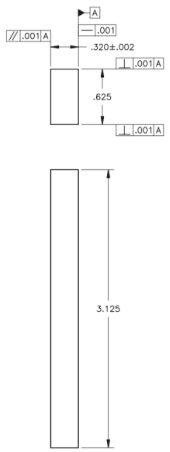
Split Disc



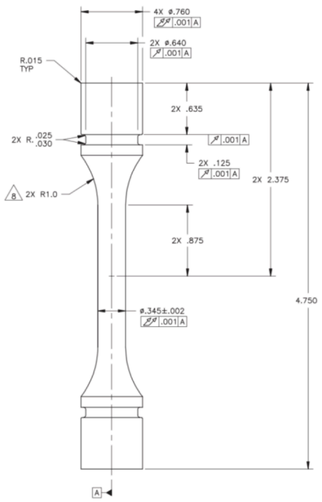
Compression



Flexural



Tensile (Dogbone)



NOTE: The split disc was not used in the tuning Vm and delta study.

Full Assessment: tuning V_m and Δ (3/3) RESULTS FOR NBG-18

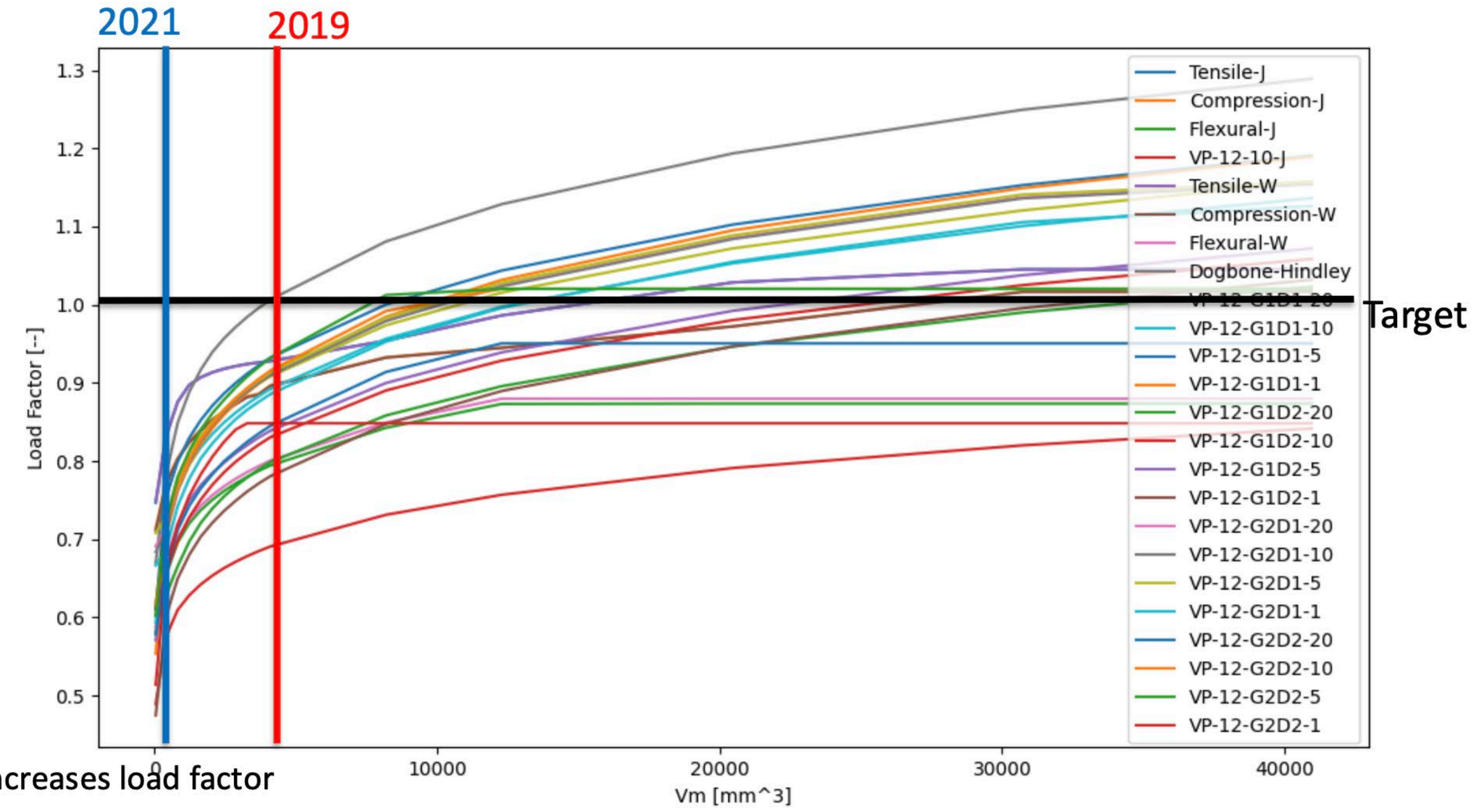
If Vm is too large, the Load Factor will be greater than 1, indicating that the methodology is non-conservative in estimating the median probability of failure.

If Vm is too small, the Load factor is less than 1, indicating the methodology is conservative in estimating the median probability of failure.

Across all grades, the dogbone gauge volume under high uniform tensile stress provides the most limiting Vm.

This work suggests Vm is not a material property, but rather based on the diameter guidelines found in ASTM C749 standard for making the dogbones. Previous Code rules based Vm on grain size and fracture toughness.

On-going work is to understand the volume effects on the strength of graphite, to ensure weakest link theory is satisfied.



Δ = 0.07

Increasing Vm increases load factor

"Hindley Dogbone" is limiting

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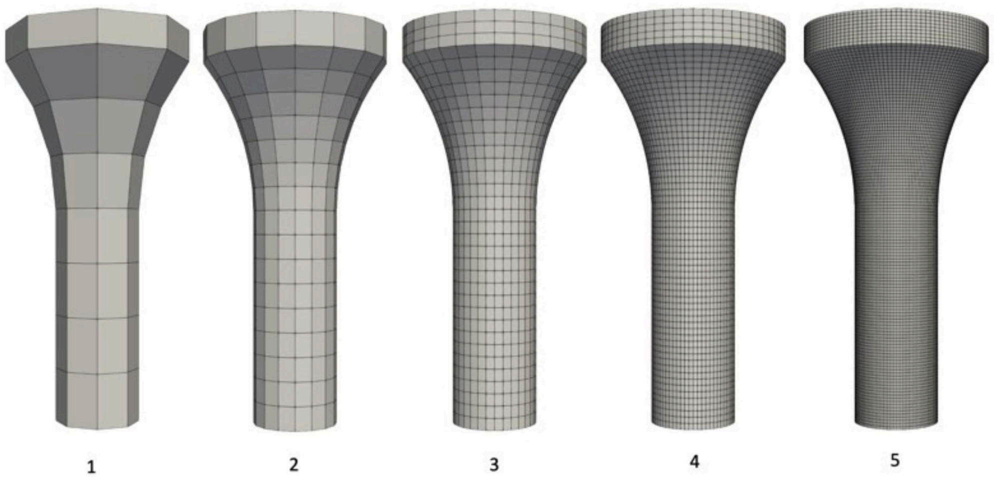
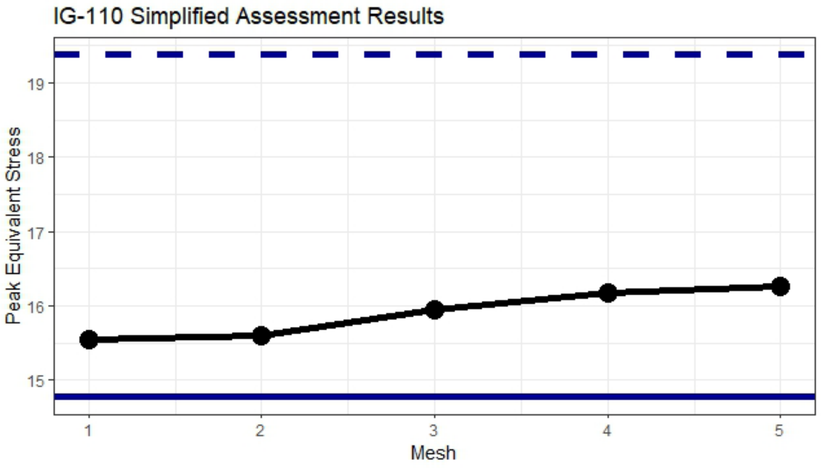
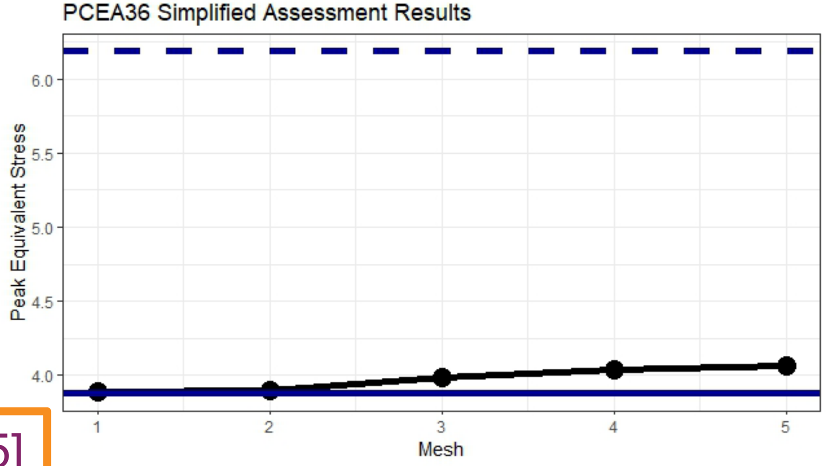
Andrea Mack
Joseph Bass

Mesh refinement (1/2) SIMPLIFIED ASSESSMENT







Copied from [5]

[5] Mack, A., Hoffman, W., Bass, J., & Windes, W. (2023). Finite element model mesh refinement effects on qualification of nuclear grade graphite components. *Proceedings of the ASME 2023 Pressure Vessels & Piping Conference*. PVP2023-107369.

Check 1: Membrane stress cannot exceed S_g
- Stresses in component were scaled such that the membrane stress = S_g. Membrane stress was unaffected by mesh refinement in this application.

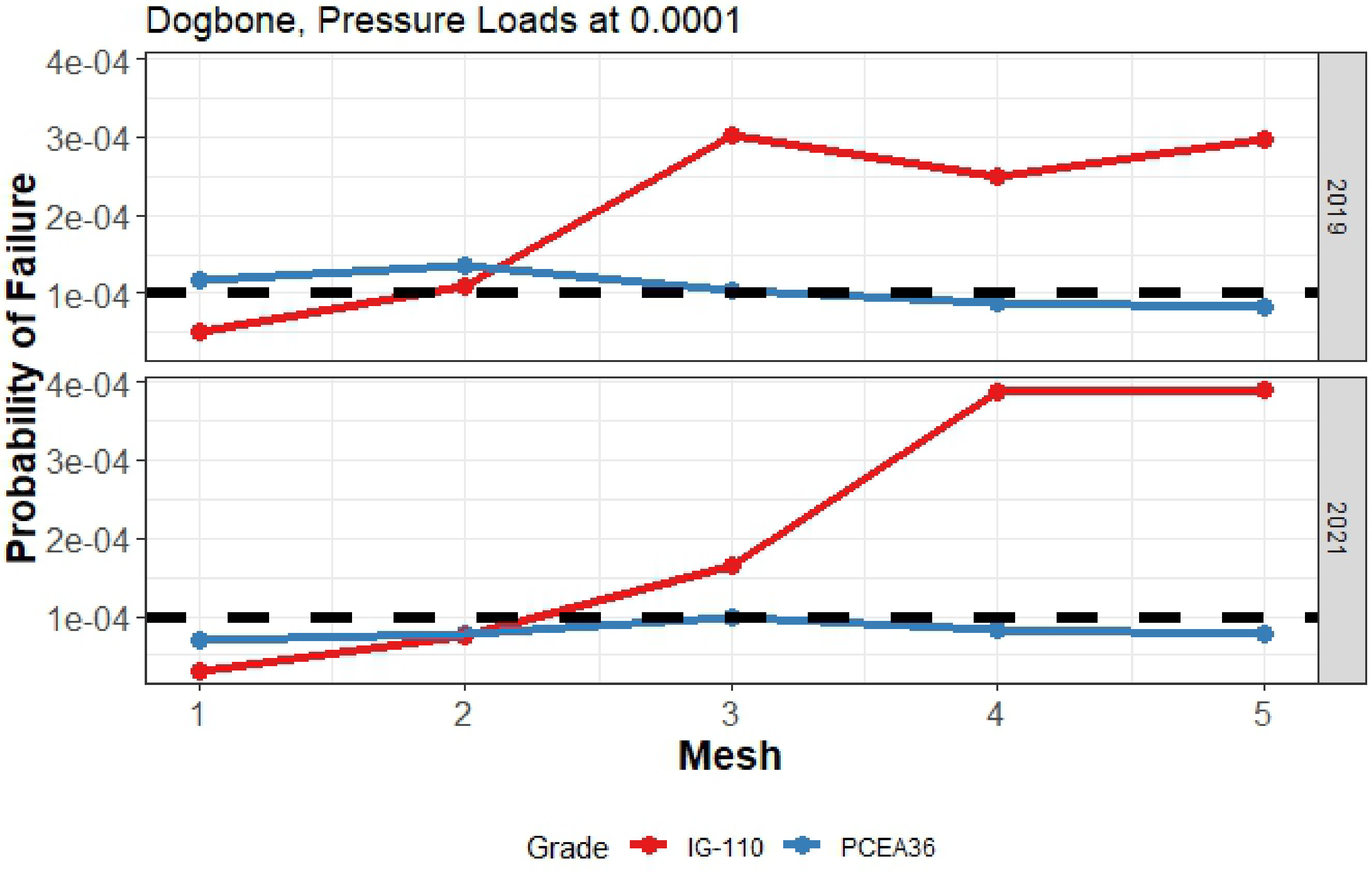
Check 2: Peak Equivalent Stress cannot exceed $S_g * R_{tf}$
- At the scaled stress state all PES are below $R_{tf} * S_g$, regardless of mesh refinement

Grade	Membrane Stress Check	Peak Equivalent Stress Check
IG-110	PASS	PASS
PCEA36	PASS	PASS

Grade/Metric	$S_g(10^{-4})$ MPa	$R_{tf} * S_g(10^{-4})$ MPa
IG-110	14.77	19.386
PCEA36	3.877	6.188

Mesh refinement (2/2) FULL ASSESSMENT

Mesh refinement **did** affect the full assessment results in this application.



IG-110	V_m	Δ
2019	$0.008\ mm^3$	0.07
2021	$14.07\ mm^3$	0.2
PCEA	V_m	Δ
2019	$512\ mm^3$	0.07
2021	$572.1\ mm^3$	0.2

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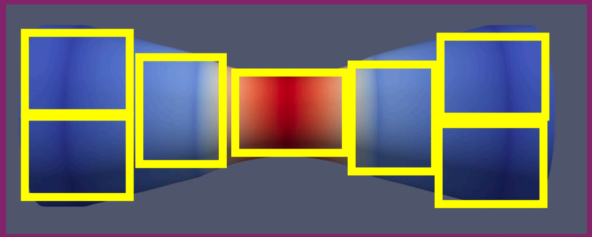
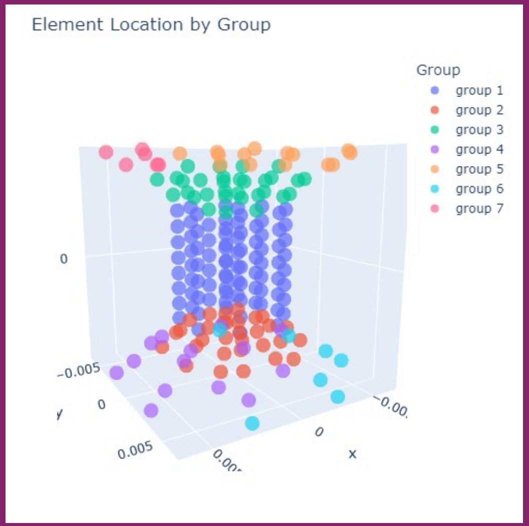
IG-110	Percent Change		
	PES	POF 2019	POF 2021
Meshes 2-4	3.7%	298.5%	210.3%
Meshes 4-5	0.5%	30.5%	15.7%
PCEA36	**PES**	**POF 2019**	**POF 2021**
Meshes 2-4	3.6%	33.4%	66.8%
Meshes 4-5	0.5%	-0.7%	-0.6%

The study done for PVP was limiting in that it only used simple geometries and it suggests repeating the study when Code rules are agreed upon for geometries in more complex stress states.

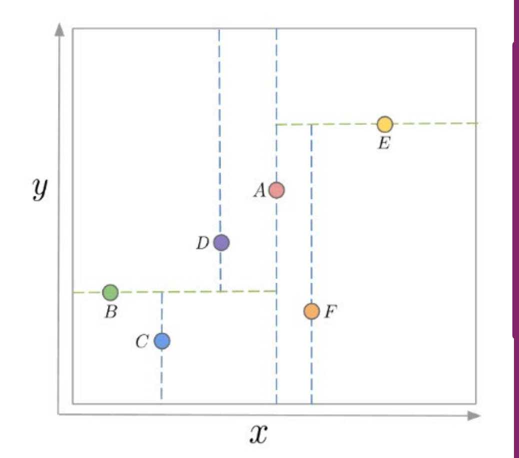
Full Assessment: Location KD-TREE

A way to quickly look up K-Dimensional objects based on their location in space.





1. Create a K-D Tree



Source: Introduction to K-D Trees | Baeldung on Computer Science

2. Group using the following algorithm

Limitations of KD tree:
- Algorithm is not efficient for fine meshes
- Produces non-spatially connected groups in geometries aside from the dogbone

Moving forward, suggestion to look into agglomerative clustering.

Find element (A) with the highest stress

↓

Locate element A's nearest neighbors (B)

↓

Calculate delta and Vm for the group

← Conditions not met

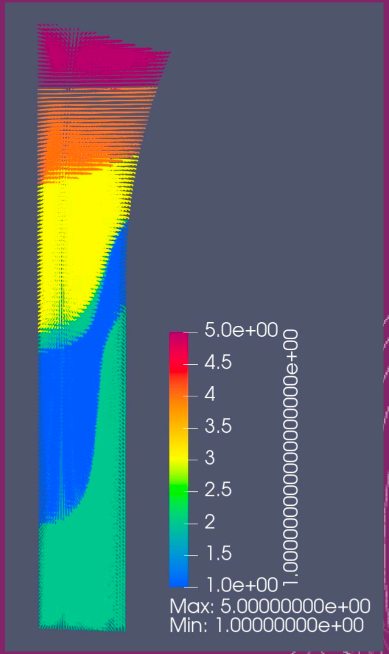
↓

Locate element A's nearest neighbor excluding last nearest neighbor added to group (→ back to Calculate delta and Vm for the group)

→ Conditions met

↓

Create group with elements and add used elements to a list so that there are no repeats (truncating is not available without remaking the KD-Tree) (→ back to Find element (A) with the highest stress)



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Ben Spencer
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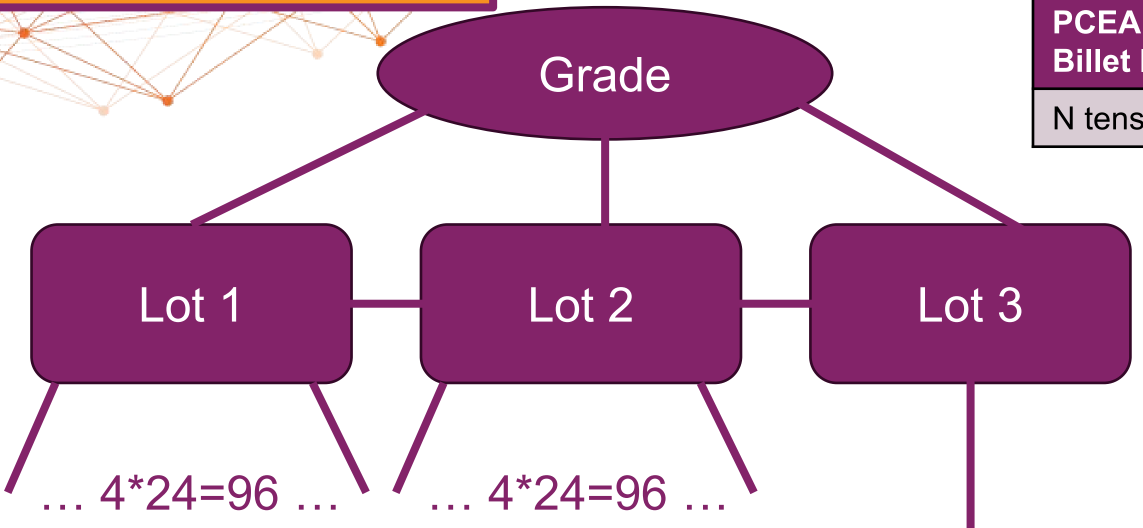
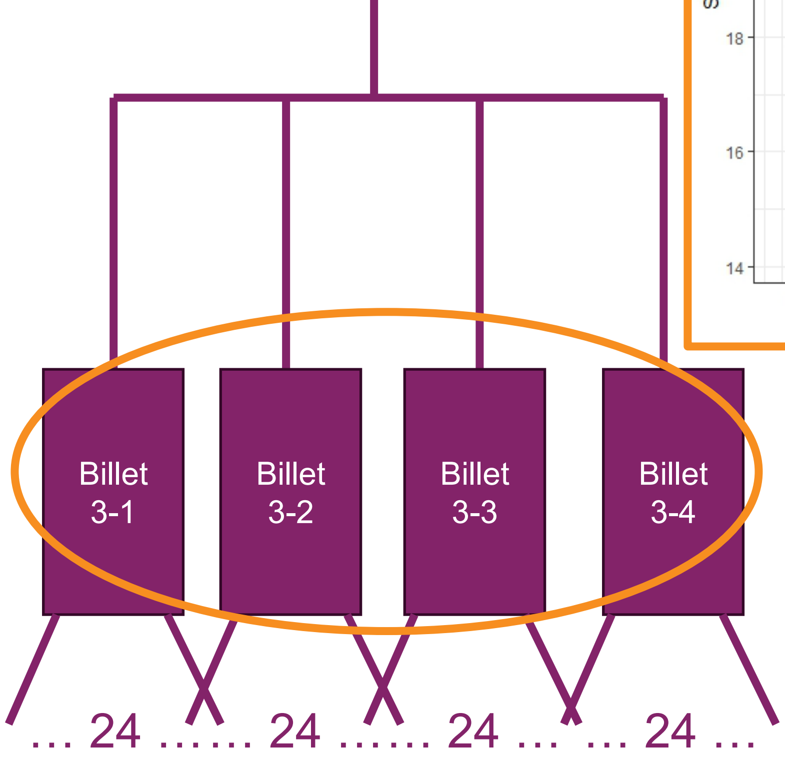
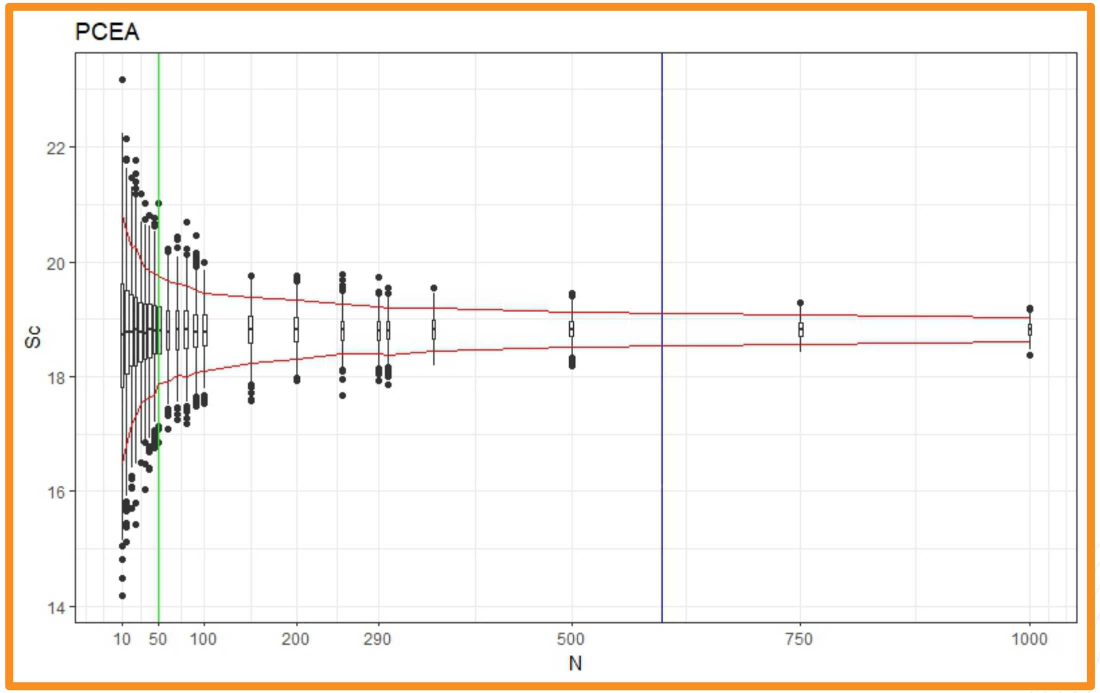
Gwennael Bieranart

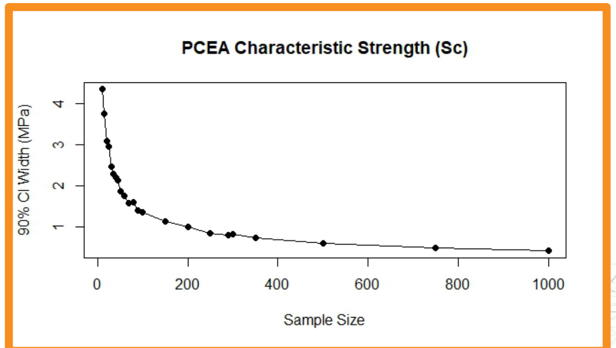
Full Assessment: Sample size requirements

PCEA Billet ID	01D3-36	02S8-5	02S8-7	01S8-9	01S8-11	Total
N tensile	127	120	192	160		599









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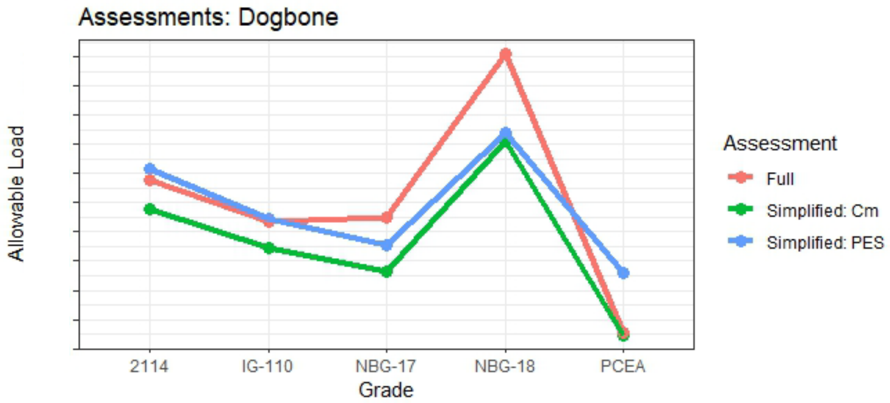
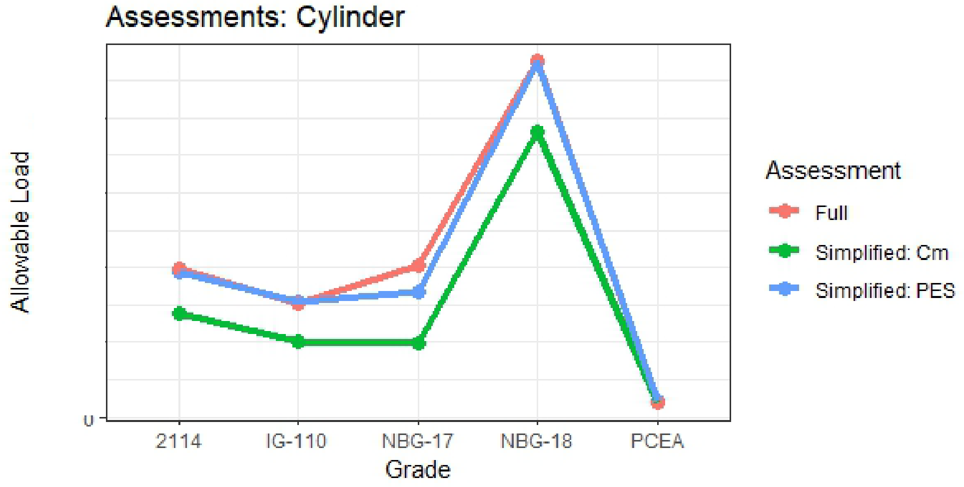
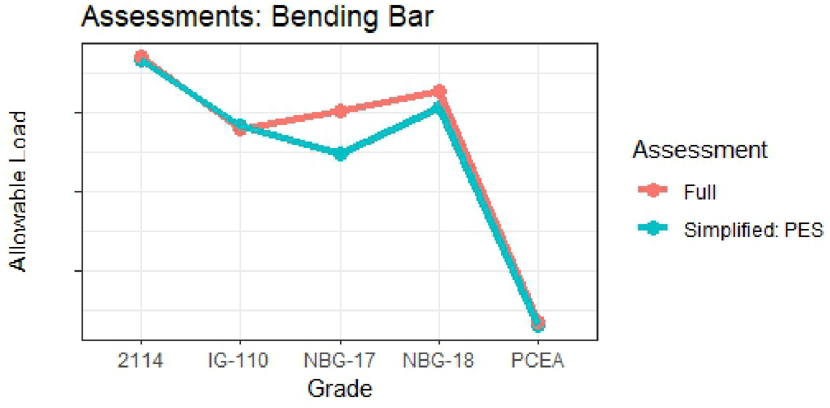
Results suggest N=288 may be adequate, but the data used do not meet the sampling requirements of HHA-II-4100 and do not account for the between-lot variability.

Margin (1/2)







PROJECT TEAM: INL
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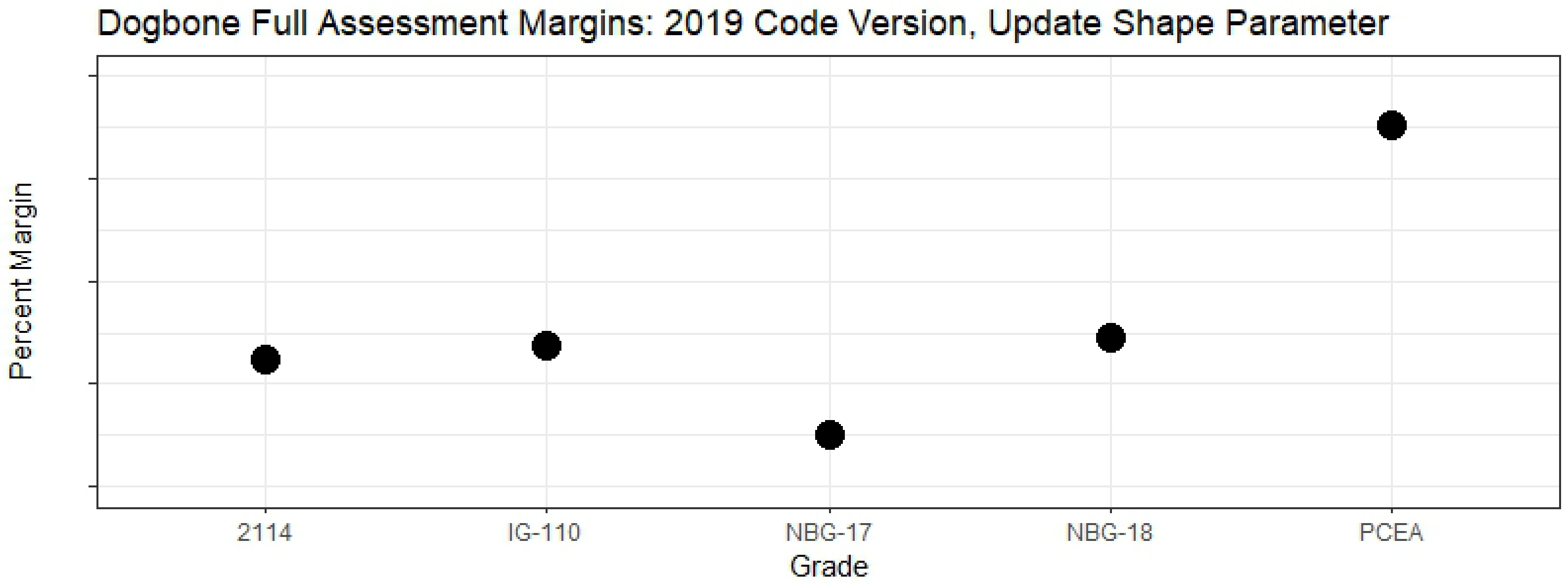
Let the margin (M) be defined as:

$$M = \left(1 - \frac{l_{0.0001}^{LB}}{\hat{S}_c}\right)$$

Where:

$l_{0.0001}^{LB}$ is the allowable load, which gives a POF=0.0001 when applying the full/simplified assessments

$\hat{S}_c$ is the characteristic strength estimated from the experimental data



Margin: Future Work (2/2)

- Future work will involve breaking down the margin into its components, to understand the proportion of margin added in at each step.
- Margin is added at the following steps of the full assessment:
  - V_m
    - V_m is tuned such that the load that results in the full assessment POF=0.5 when using MLEs perfectly matches the experimental median for the "most limiting" geometry, which has been determined to be a purely tensile specimen with minimum gauge length.
    - The most limiting V_m will also be used for all other geometries, but will result in conservative results.
    - Define $l_{0.632}^{MLE}$ as the full assessment load which gives a POF=0.632 when using the parameter MLE's.
    - Define: $M(V_m) = 1 - \frac{l_{0.632}^{MLE}}{\hat{S}_c}$
  - LB
    - Margin is added into the full assessment by using 95% one-sided Weibull parameter lower bounds to account for sampling uncertainty in the parameter estimates.
    - Define $l_{0.632}^{LB}$ as the full assessment load which gives a POF=0.632 when using the parameter LB's
    - Define: $M(LB) = 1 - \frac{l_{0.632}^{LB}}{\hat{S}_c}$
  - SRC
    - Margin is again added into the full assessment by using the POF limit allowed by the component's structural reliability class (SRC). For these purposes, we are using the SRC limit of 0.0001.
    - Define $l_{0.0001}^{MLE}$ as the full assessment load which gives a POF=0.0001 when using the parameter MLE's
    - Define: $M(10^{-4}) = 1 - \frac{l_{0.0001}^{MLE}}{\hat{S}_c}$
- Then the total margin (M) can be broken into parts as:

$$\boldsymbol{M = M(V_m) + M(LB) + M\left(10^{-4}\right) - [2 * M(V_m)]}$$

Note that M(LB) and M(10^-4) both contain M(V_m), there is no easy way to remove it, so it must be subtracted off.

$$\boldsymbol{M = M(LB) + M\left(10^{-4}\right) - M(V_m)}$$

We can use this equation to understand where the most amount of conservatism is coming from in the full assessment.

Simplified Assessment: Stress terminology & R_{tf}

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PROBLEM

- There are two checks for the simplified assessment:
  - Membrane stress check
  - "Peak" stress check
- The second check raises the allowable stress by a factor of R_{tf}. There is concern R_{tf} makes the simplified assessment less conservative than the full.

ACCOMPLISHMENTS

- Determined R_{tf} effectively raises the 10^{-4} tensile limit to a 10^{-4} flexural limit
- Showed by comparison with tensile, compressive, and flexural experimental data that the simplified method is conservative as long as the two criteria are being met (if updating the shape parameter in the full assessment)
- Determined normal stress-based failure criteria are overly conservative for compressive load cases

ACTION ITEMS

- R23-473: will change stress terminology to MDE not normal stress
- Further understand the implication of the use of the Rtf on the method's conservatism and the possibility to use a single failure criterion

Schedule

Subtask groups meet every other week between the TG meetings.

Jan. '23

NRC meeting: R_{tf}

April '23

TG meeting: Mesh refinement and shape parameter update

May '23

ASME Code Week: *Simplified assesment* and tuning V_m and Δ

July '23

TG meeting: Disparate flaw distribution and shape parameter update

Aug. '23 ASME

ASME Code Week: Simplified assessment and tuning V_m and Δ

Goal: Code rule changes by 2025 BPVC

Conclusions

- Disparate flaw distribution
- Tuning V_m and Δ
- Feasibility study on incorporating location in full assessment
- Margin 2019 Code rules
- Margin 2025 Code rules
- Mesh refinement study more complex geometry
- Volume effects on strength of graphite
  - When is weakest link theory valid
  - Effect of scaling strength distributions to realistic size components
  - Experimental validation of full assessment using a realistic component in a complex stress state
- Adequate sample size to characterize graphite tensile strength distribution

KEY FOR COMPLETION TARGETS:
Purple: Dec. 2023, technical work complete
Dec. 2024, results submitted for publication and records accepted
Yellow: 2024 technical work
Red: Missing information, not able to complete

Concern in meeting targets: all subtasks are related, so many cannot be completed until all are completed.

Goal:

All Design changes to code rule approved for the **2025 version** of the Boiler and Pressure Vessel Code (BPVC)